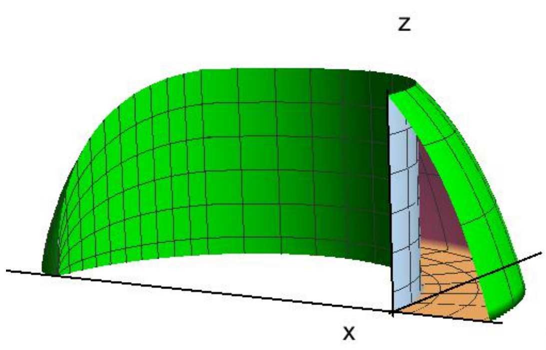
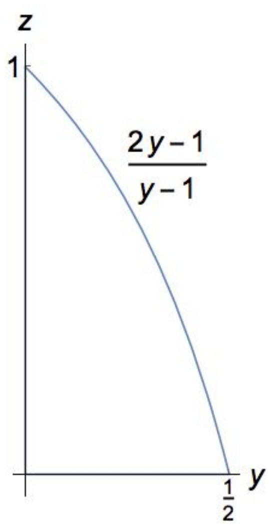
Math 20E Review Problems for Sections 8.2, 8.3, and 8.4

Summer Session II, 2015

1. Pictured is a cave whose only opening is the region in the y-z plane bounded by the y-axis, the z-axis, and the curve $z = \frac{2y-1}{y-1}$. Consider the walls and floor of the cave as a surface S oriented by the inward pointing normal.





(a) Indicate the orientation of ∂S by drawing arrows on the above right figure.

(b) Let

$$\vec{\mathbf{F}}(x, y, z) = zy \arctan(y^2)\, \vec{\mathbf{i}} + zy(y - 1 + xe^{\sin(xz)})\, \vec{\mathbf{j}} + zy^2(y-1)^3\, \vec{\mathbf{k}}$$

Use Stokes' Theorem to evaluate $\iint_S (\nabla \times \vec{\mathbf{F}}) \cdot d\vec{\mathbf{S}}$.

2. Let $\vec{\mathbf{F}}(x, y, z) = xze^{-y}\, \vec{\mathbf{i}} + (ze^{-y} + y \sin z)\, \vec{\mathbf{j}} + (\cos z + \frac{x^2}{2} e^{-y})\, \vec{\mathbf{k}}$.

(a) Find curl $\vec{\mathbf{F}} = \nabla \times \vec{\mathbf{F}}$ and div $\vec{\mathbf{F}} = \nabla \cdot \vec{\mathbf{F}}$.

(b) Is there a vector field $\vec{\mathbf{G}} : \mathbb{R}^3 \to \mathbb{R}^3$ such that $\vec{\mathbf{F}} = \nabla \times \vec{\mathbf{G}}$?

(c) Is there a function $f : \mathbb{R}^3 \to \mathbb{R}$ such that $\vec{\mathbf{F}} = \nabla f$? If so, find f and evaluate $\int_{\boldsymbol{c}} \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}}$ where $\boldsymbol{c}$ is the curve parametrized by

$$\boldsymbol{c}(t) = \begin{cases} (t\cos t,\ t\sin t,\ t), & \text{for } \pi \le t \le 2\pi; \\ (t(\sin t + 1),\ t(\cos t - 1),\ t), & \text{for } 2\pi \le t \le 3\pi. \end{cases}$$

3. Redo the previous problem for

$$\vec{\mathbf{F}}(x, y, z) = (yz + y)\, \vec{\mathbf{i}} + (xz + x)\, \vec{\mathbf{j}} + (xy + 1)\, \vec{\mathbf{k}}$$

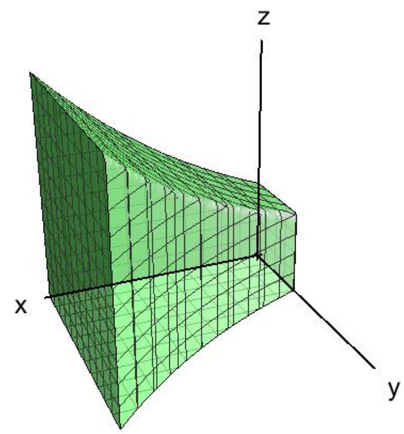
(This was formerly a different vector field, but it was way too algebraically intense, for which I apologize. I will post the answer for the original $\vec{\mathbf{F}}$ as well, in case you did it already).

4. Consider the solid

$$W = \{(x, y, z) \in \mathbb{R}^3 : 0 \le x \le 1,\ 0 \le y \le e^x,\ 0 \le z \le e^x\}$$

and let $S = \partial W$ be the boundary of W. Let

$$\vec{\mathbf{F}}(x, y, z) = -2ze^{-2x}\ \vec{\mathbf{i}}\ + \arctan(e^{xz})\ \vec{\mathbf{j}}\ + 3z(1 + y)^{1/2}\ \vec{\mathbf{k}}$$



(a) Find $\nabla \cdot \vec{\mathbf{F}}$.

(b) Use Gauss' Divergence Theorem to evaluate $\iint\limits_S \vec{\mathbf{F}} \cdot d\vec{\mathbf{S}}$.